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Discrete singularities method in problems of seismic and impulse impacts on reservoirs

D. V. Krutchenko[1)], E. A. Strelnikova[1),2)], Yu. S. Shuvalova[3)]
Институт проблем машиностроения им. А.Н.Подгорного НАНУ, Украина,
Харьковский национальный университет им. В.Н. Каразина
Украинский государственный университет железнодорожного транспорта

A numerical method is proposed to simulate impulse and seismic effects on storage with a liquid. The liquid is supposed to be ideal, incompressible, and its current is irrotational. The fluid pressure satisfies the Cauchy-Lagrange integral. To determine it, we obtain a system of integral equations. Its numerical solution is obtained by the method of boundary elements. The eigenvalues and forms of fluid vibrations are obtained. The proposed method made it possible to estimate the level of the free surface at a sudden applied load.

Key words: *резервуары с жидкостью, метод интегральных уравнений, свободные и вынужденные колебания*

Предложен численный метод, для моделирования импульсного и сейсмического воздействия на хранилища с жидкостью. Предполагается что жидкость идеальная, несжимаемая, а её течение безвихревое. Давление жидкости удовлетворяет интегралу Коши-Лагранжа. Для его определения получена система интегральных уравнений. Её численное решение получено методом граничных элементов. Получены собственные значения и формы колебаний жидкости. Предложенный метод позволил оценить уровень свободной поверхности при внезапно приложенной нагрузке.

Ключевые слова: *резервуары с жидкостью, метод интегральных уравнений, свободные и вынужденные колебания*

Запропоновано чисельний метод для моделювання імпульсу і сейсмічної дії на сховища з рідиною. Припускається, що рідина ідеальна, нестислива, а її рух є безвихровим. Тиск рідини задовольняє інтегралу Коші-Лагранжа. Для його визначення отримана система інтегральних рівнянь. Її чисельний розв’язок отримано методом граничних елементів. Отримано власні значення і форми коливань рідини. Запропонований метод дозволив визначити рівень вільної поверхні при раптово прикладеному навантаженні.

Ключові слова: *резервуары с жидкостью, метод интегральных уравнений, свободные и вынужденные колебания*

1. Problem statement and its topicality

Containers and tanks for storage of oil, flammable and poisonous liquids are widely used in various fields of engineering practice, such as aircraft industry, chemical and oil and gas industry, power engineering, transport. These tanks usually operate at raised technological loadings and they are filled with oil, flammable or toxic agents. As a result of sudden action of earthquakes, shockwaves, other force majeur circumstances the liquid stored in tanks may be exposed to intensive sloshing.

Sloshing is a phenomenon observed in a number of industrial facilities: in containers for storage of the liquefied gas, oil, fuel tanks, in tanks of cargo tankers. It is known that partially filled tanks are affected by especially intensive sloshing. It can lead to high pressure on tank walls, to destruction of structures or loosing stability, and to leakage of dangerous contents, that in turn, can lead to serious ecological consequences.

The analysis of research devoted to the problems of liquid sloshing in tanks is given in R. A. Ibrahim's works [1,2]. Note also the works devoted to liquid sloshing in cylindrical tanks under seismic loadings [3-5].

In this paper the problem concern with liquid vibrations in a shell of revolution is considered. We designate a moistened shell surface by S_1, and a free surface by S_0. Suppose the Cartesian coordinate system 0xyz is connected with the shell, the liquid free surface S_0 coincides with the x0y plane at the state of rest (fig. 1)

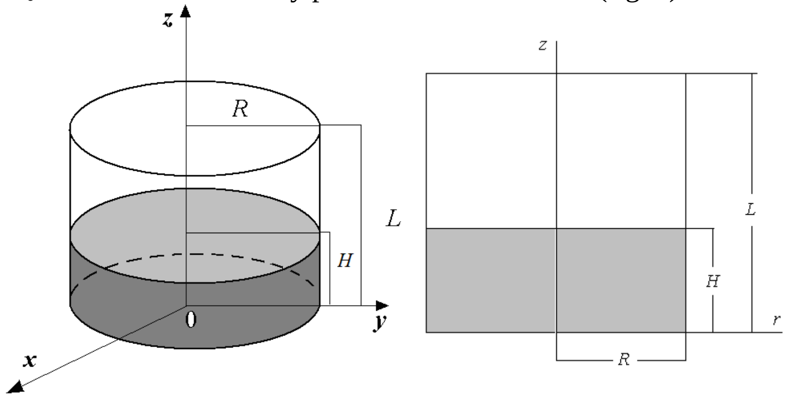

Fig. 1. Fluid-filled cylindrical shell and its sketch.

Suppose that liquid is ideal and incompressible one and its movement started from the state of rest is irrotational. Then there exist a liquid velocity potential Φ

$$V_x = \frac{\partial\Phi}{\partial x}; V_y = \frac{\partial\Phi}{\partial y}; V_z = \frac{\partial\Phi}{\partial z},$$

satisfying to Laplace's equation.

We determine pressure p upon shell walls from the linearized Cauchy-Lagrange's integral by the following formula

$$p = -\rho_l\left(\frac{\partial\Phi}{\partial t} + gz\right) + p_0 + a_s(t)x,$$

Here Φ is the velocity potential, g is the acceleration of gravity, z is a point vertical coordinate in the liquid, ρ_l is the liquid density, p_0 is an atmospheric pressure, $a_s(t)$ is a function characterizing external influence (a horizontal seism or an impulse).

On the free surface of liquid the following conditions have to be satisfied:

$$\left.\frac{\partial\Phi}{\partial n}\right|_{S_0} = \frac{\partial\zeta}{\partial t}; \quad \left.p - p_0\right|_{S_0} = 0,$$

where the function ζ describes the form and location of the free surface.

Thus, for the potential we have the following boundary problem

$$\nabla^2\Phi = 0; \left.\frac{\partial\Phi}{\partial \mathbf{n}}\right|_{S_1} = 0; \left.\frac{\partial\Phi}{\partial n}\right|_{S_0} = \frac{\partial\zeta}{\partial t}; \quad \left.p - p_0\right|_{S_0} = 0; \left.\frac{\partial\Phi}{\partial t} + g\zeta + a_s(t)x\right|_{S_0} = 0.$$

Having determined the velocity potential Φ and the function ζ, we establish height of raising of the free surface and determine the liquid pressure upon shell walls.

2. The mode superposition method.

Consider the potential **Φ** in the next form

$$\Phi = \sum_{k=1}^{M} \dot{d}_k \varphi_k \ . \tag{1}$$

For functions φ consider the following boundary problems:

$$\nabla^2 \varphi_k = 0 \ , \ \left. \frac{\partial \varphi_k}{\partial \mathbf{n}} \right|_{S_1} = 0 \ , \tag{2}$$

$$\left. \frac{\partial \varphi_k}{\partial \mathbf{n}} \right|_{S_0} = \frac{\partial \varsigma}{\partial t}; \quad \frac{\partial \varphi_k}{\partial t} + g\zeta = 0 \ . \tag{3}$$

Differentiate the second relation in (3) on and substitute it in the received equality $\frac{\partial \varsigma}{\partial t}$ from the first relation. Further we present the functions φ_k in the next form $\varphi_k(t,x,y,z) = e^{i\chi_k t}\varphi_k(x,y,z)$. We come to an eigenvalue

$$\frac{\partial \varphi_k}{\partial n} = \frac{\chi_k^2}{g}\varphi_k \ . \tag{4}$$

As an equation for the free surface we obtain expression

$$\zeta = \sum_{k=1}^{M} d_k \frac{\partial \varphi_k}{\partial n} \ . \tag{5}$$

In cylindrical coordinates system we have following expressions

$$\varphi_k(r,z,\theta) = \varphi_k(r,z)\cos\alpha\theta \tag{6}$$

Here α is a harmonica number. Thus, frequencies and modes of free vibrations are considered separately for different α.

We present φ as of potentials of a simple and double layers [5]

$$2\pi\varphi(P_0) = \iint_S \frac{\partial \varphi}{\partial n} \frac{1}{|P - P_0|} dS - \iint_S \varphi \frac{\partial}{\partial n} \frac{1}{|P - P_0|} dS \ . \tag{7}$$

Here $S = S_1 \cup S_0$; points P and P_0 belong to surface S. By $|P - P_0|$ we denote the Cartesian distance between points P and P_0.

With boundary conditions (2),(3), we come to system of the integral equations in the form [6,7]:

$$\begin{cases} 2\pi\varphi_1 + \iint\limits_{S_1} \varphi_1 \frac{\partial}{\partial n}\left(\frac{1}{r}\right) dS_1 - \frac{\kappa^2}{g} \iint\limits_{S_0} \varphi_0 \frac{1}{r} dS_0 + \iint\limits_{S_0} \varphi_0 \frac{\partial}{\partial z}\left(\frac{1}{r}\right) dS_0 = 0, \\ -\iint\limits_{S_1} \varphi_1 \frac{\partial}{\partial n}\left(\frac{1}{r}\right) dS_1 - 2\pi\varphi_0 + \frac{\kappa^2}{g} \iint\limits_{S_0} \varphi_0 \frac{1}{r} dS_0 = 0. \end{cases} \tag{8}$$

Here for convenience we denote values of potential on the free surface by φ_0 and by φ_1 on the shell walls.

We look for the solution of system (8) in form (9).

Previously, having integrated equation (8) by the variable θ, we obtain the following system of one-dimensional singular equations.

$$2\pi\varphi(z_0)+\int_\Gamma \varphi(z)Q(z,z_0)r(z)d\Gamma-\int_0^R q(\rho)\Psi(P,P_0)\rho d\rho=\int_\Gamma w(z)\Psi(P,P_0)r(z)d\Gamma_1;\, P_0\in S_1, \quad (9)$$

$$\int_\Gamma \varphi(z)Q(z,z_0)r(z)d\Gamma-\int_0^R q(\rho)\Psi(P,P_0)\rho d\rho=\int_\Gamma w(z)\Psi(P,P_0)r(z)d\Gamma_1;\; P_0\in S_0.$$

Here

$$Q(z,z_0)=\frac{4}{\sqrt{a+b}}\left\{\frac{1}{2r}\left[\frac{r^2-r_0^2+(z_0-z)^2}{a-b}\mathrm{E}_\alpha(k)-\mathrm{F}_\alpha(k)\right]n_r+\frac{z_0-z}{a-b}\mathrm{E}_\alpha(k)n_z\right\};$$

$$\Psi(P,P_0)=\frac{4}{\sqrt{a+b}}\mathrm{F}_\alpha(k);\; \mathrm{E}_\alpha(k)=(-1)^\alpha\left(1-4\alpha^2\right)\int_0^{\pi/2}\cos 2\alpha\psi\sqrt{1-k^2\sin^2\psi}\,d\psi\,;$$

$$\mathrm{F}_\alpha(k)=(-1)^\alpha\int_0^{\pi/2}\frac{\cos 2\alpha\psi d\psi}{\sqrt{1-k^2\sin^2\psi}}\,;\quad a=\rho^2+\rho_0^2+\left(z^*-z_0\right)^2;\quad b=2\rho\rho_0;\; k^2=\frac{2b}{a+b}.$$

To define potentials φ_k we use representation (9) and introduce next integral operators:

$$A\psi_1=2\pi\psi_1+\iint_{S_1}\psi_1\frac{\partial}{\partial n}\frac{1}{r(P,P_0)}dS_1;\; B\psi_0=\iint_{S_0}\psi_0\frac{1}{r}dS_0;\; C\psi_0=\iint_{S_0}\psi_0\frac{\partial}{\partial z}\left(\frac{1}{r}\right)dS_0\,;$$

$$D\psi_1=-\iint_{S_1}\psi_1\frac{\partial}{\partial n}\frac{1}{|P-P_0|}dS_1\,;\; F\psi_0=\iint_{S_0}\psi_0\frac{1}{r}dS_0\,. \quad (10)$$

Then the boundary value problem (2)-(5) takes the form

$$A\psi_1=\frac{\kappa^2}{g}B\psi_0-C\psi_0\,;\;\; P_0\in S_1\,; \qquad D\psi_1=2\pi E\psi_0-\frac{\kappa^2}{g}F\psi_0\,; \qquad P_0\in S_0\,.$$

After excluding function ψ_1 from these relations we obtain the following eigenvalue problem

$$(DA^{-1}C+E)\psi_0-\lambda(DA^{-1}B+F)\psi_0=0;\quad \lambda=\frac{\chi^2}{g}$$

Its solution gives natural modes and frequencies of liquid sloshing in rigid tank.

Evaluation of integral operators in (10) is carried out by the method proposed in [8-10].

3. Reducing the dynamic problem to a system of differential equations.

Having defined the basic functions φ_k, substitute them in expressions for velocity potential (1) and for the free surface elevation (5). Then substitute the received relations in the boundary condition on the free surface

$$\left. \frac{\partial \Phi}{\partial t} + g\zeta + a_s(t)x \right|_{s_0} = 0 .$$

As in cylindrical system of coordinates there is $x = r\cos\theta$, we will be interested only in the first harmonica, i.e. in a formula (6) we only consider α=1. We come to the following equation on the surface S_0

$$\sum_{k=1}^{M} \ddot{d}_k \varphi_k + g\sum_{k=1}^{M} d_k \frac{\partial \varphi_k}{\partial n} + a_s(t)r = 0 .$$

Due to validity of relation (4) on the surface S_0 the equality given above takes the form

$$\sum_{k=1}^{M} \ddot{d}_k \varphi_k + \sum_{k=1}^{M} \chi_k^2 d_k \varphi_k + a_s(t)r = 0 . \qquad (11)$$

Accomplishing the dot product of equality (11) by $\varphi_l \left(l = \overline{1, M}\right)$ and having used orthogonality of own modes, we receive the system of ordinary differential equations of the second order

$$\ddot{d}_k + \chi_k^2 d_k + a_s(t)F_k = 0; \quad F_k = \frac{(r, \varphi_k)}{(\varphi_k, \varphi_k)}; \quad k = \overline{1, M} . \qquad (12)$$

Suppose that before applying the horizontal impulse the tank was at the state of rest. Then we have to solve (12) under zero initial conditions. The operational method is applied here to the solution of system (12).

The following values for coefficients $d_k(t), \quad k = \overline{1, M}$ are obtained:

$$d_k(t) = \begin{cases} \dfrac{1}{\chi_k^2} - \dfrac{1}{\chi_k^2}\cos(\chi_k t) \quad 0 \le t \le T \\ \dfrac{1}{\chi_k^2} - \dfrac{1}{\chi_k^2}\cos(\chi_k t) - \dfrac{1}{\chi_k^2} + \dfrac{1}{\chi_k^2}\cos\chi_k (t - T) \;\; t > T \end{cases}$$

Substituting these coefficients in relation (5), one can obtain the time-history of the free surface elevation.

4. Analysis of numerical results.

We will consider the cylindrical shell with a flat bottom partially filled with the liquid. The tank parameters are following: radius is R = 1 m, thickness is h= 0.01m, length is L = 2 m, filling level is H =0.8m.

For carrying out the calculations we accepted different numbers of the basic functions.

Fig. 2 shows the time-history of the free surface elevation in the point B with r=1.5 (see fig. 1). Here the only one (M=1) basic function is used in (5).

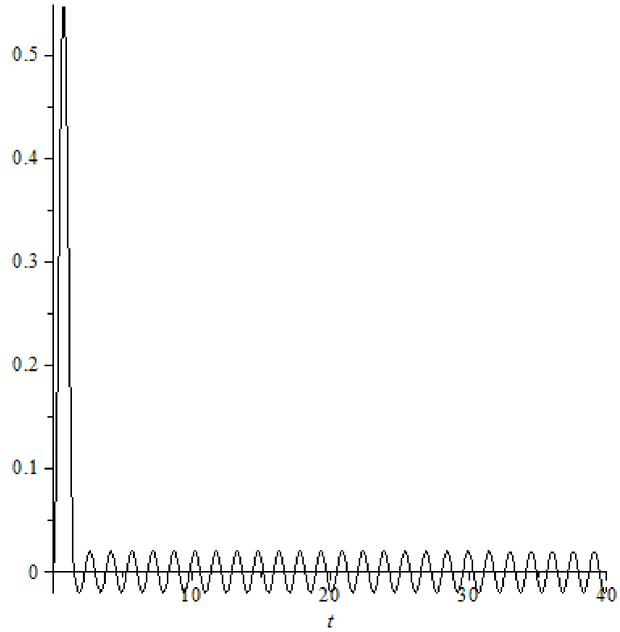

Fig.2. Time –history of the free surface at impulse loading, M=1.

On fig. 3 the free surface elevation in the point B with r=1.5 point depending on time is shown. Here we use three basic functions (M=3 in (5)).

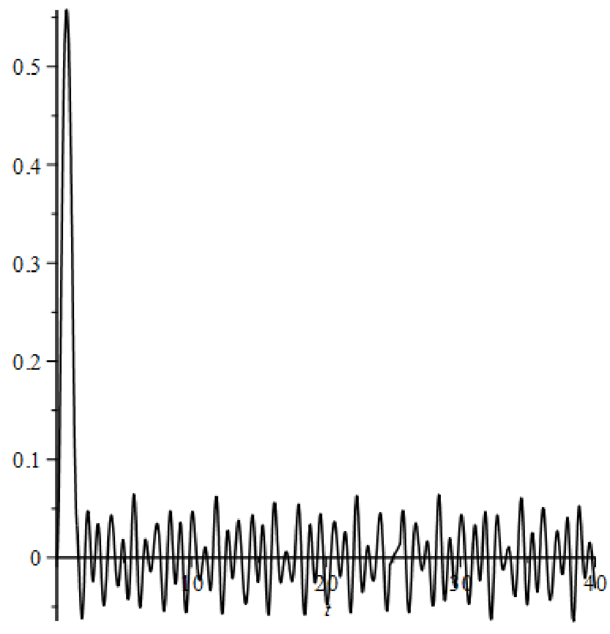

Fig.3. Time –history of the free surface at impulse loading, M=3.

Further increasing in number of basic functions didn't lead to essential change of results.

Conclusion

The developed method allows us to estimate the level of the free surface elevation at suddenly enclosed loadings. This approach will be easy generalized for elastic tanks with elastic baffles. The geometry of tank also can be easy changed, so the results will be obtained for conical, spherical and compound shells. It will allow to make recommendations about installation of protective elements (covers, partitions).

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