

# A statistical method for predicting the eccentric load capacity of rectangular concrete filled steel tubular columns

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**Abstract.** The article deals with the integrated approach to the study of the behaviour of rectangular CFST columns under eccentric compression. Such an approach includes the development of methods for assessing the magnitude of the carrying capacity, assessing the degree of reliability and credibility of the obtained results, as well as studying the nature of the development of columns deformations at various stages of loading. The authors developed a mathematical model for calculation of columns carrying capacity under eccentric compression based on statistical methods. Substantial amount of experimental data collected by the world leading laboratories enabled obtaining a regression dependence of the columns carrying capacity that takes into account the impact of the physical and geometric characteristics of such structures. High degree of model confidence is confirmed by a comparative analysis with experimental results that are not involved in the development of the model, as well as with calculations performed according to Eurocode, Japanese and Chinese regulatory documents. The article presents experimental studies of the nature of deformations development on the surface of the steel shell and inside the concrete core of various lengths rectangular columns. As a result of the experimental tests, it was established that the longitudinal strains of the compressed area of the shell have the most significant impact on the bearing capacity of eccentrically compressed steel concrete samples.

## 1 Introduction

The issues of improving the quality of construction projects, reducing the cost of construction work, reducing the manufacturing cycle in the development and practical implementation of design solutions will always be relevant. This requires constant updating of the technological base of the construction industry: expanding and intensifying research, introducing advanced technologies, and applying innovative approaches in the design methodology. Many works have considered the researching of strength and reliability of Concrete Filled Steel Tubular (CFST) structures that have been acknowledged in the

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construction industry [1-9]. Different approaches were also used for predicting the columns load carrying capacity [10-15].

The purpose of this work is developing methods for assessing the carrying capacity of rectangular steel concrete columns under eccentric compression, assessing the degree of confidence and objectivity of the obtained results, and analysing the nature of the destruction of such structures. Previously, the authors already applied this technique in the study of circular and rectangular CFST columns under axial compression [16-18].

To achieve this purpose, the following tasks should be solved:

- development of a mathematical model for assessing the carrying capacity using statistical methods, which reduces the time required to calculate the value of the carrying capacity;
- analysis and verification of the results; and
- conducting experimental studies.

## 2 Regression equation

To solve the described above problem, the authors used the regression and correlation analysis based on experimental results obtained by the leading world laboratories for total 146 observations.

The considered samples have the following characteristics: shell thickness  $t \geq 2.5$  mm;  $80 \leq b \leq 300$  mm;  $99.4 \leq h \leq 300$  mm; ratio  $L/b \geq 4$ , at  $450 \leq L \leq 4910$  mm; yield strength of steel  $242 \leq \sigma_y \leq 746$  MPa; steel modulus of elasticity  $2 \times 10^5 \leq E_s \leq 2.17 \times 10^5$  MPa, concrete prism strength  $15 \leq f_{c,prizm} \leq 83.6$  MPa, concrete modulus of elasticity  $26.8 \leq E_c \leq 44.53$  MPa, eccentricity  $0 < e < h$ .

The resulting regression equation has the form:

$$\bar{N} = a_0 + \sum_{i=1}^5 a_i x_i = 220.745 + 0.222x_1 - 3.7 \cdot 10^{-5}x_2 + 0.0027x_3 + (1) \\ + 2 \times 10^{-5}x_4 - 14387.2x_5$$

where

$$x_1 = (b - 2t)(h - 2t - e) f_{c,prizm} / E_s, x_2 = L\sigma_y E_c, x_3 = t\sigma_y E_s, x_4 = \frac{E_c}{\sqrt{t}} L^{1.5} (2) \\ x_5 = \frac{e}{t^{1.5} \sqrt{Lh}}$$

Coefficient of determination  $R^2$  for the model is 0.982. Mean approximation error is:

$$\Delta = \frac{1}{146} \sum_{i=1}^{146} \frac{|N_{exp}^i - N_{theor}^i|}{N_{exp}^i} 100\% \approx 7.88\% (3)$$

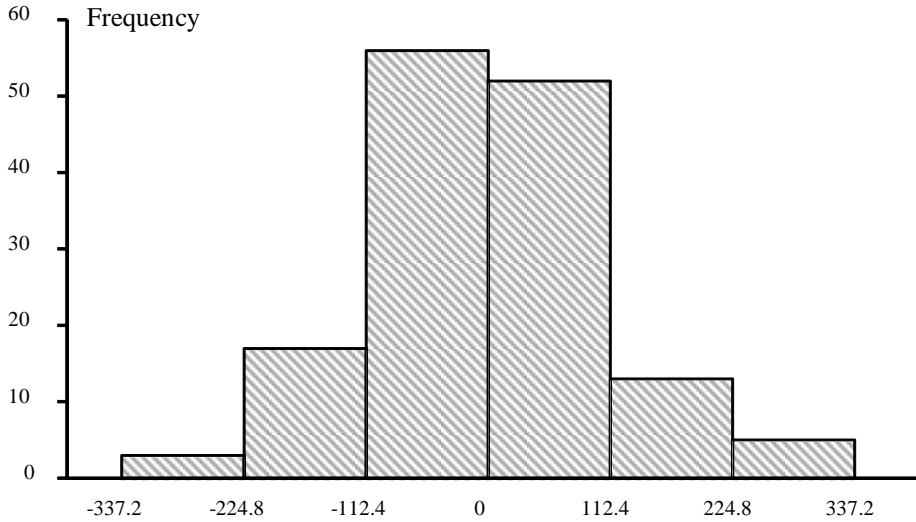
Confidence intervals for each coefficient, found with a probability of  $P = 0.95$ , show how much the obtained regression coefficients can differ from the exact values:

$$139.994 \leq a_0 \leq 301.497 \\ 0.215 \leq a_1 \leq 0.229 \\ -4.2 \times 10^{-5} \leq a_2 \leq -3.3 \times 10^{-5} \\ 0.0026 \leq a_3 \leq 0.0028 \\ 1.45 \times 10^{-7} \leq a_4 \leq 2.55 \times 10^{-7} \\ -18100.7 \leq a_5 \leq -10673.7$$

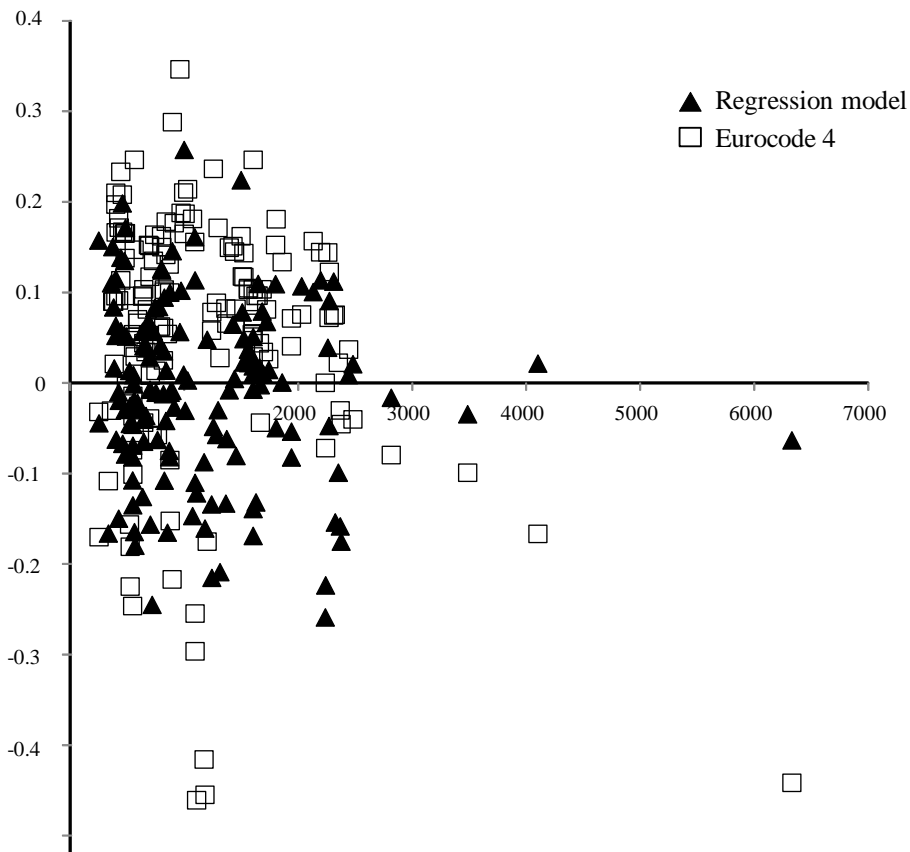
The correspondence of the obtained model to the experimental data, as well as the significance of the regression parameters, is confirmed by the Fisher and Student criteria, respectively.

Fig. 1 shows a histogram of residuals  $N_{theor} - N_{exp}$ . According to the Pearson criterion, the obtained sample at a significance level of  $\alpha = 0.05$  corresponds to the normal distribution law with zero expected value, and all values fall into the interval  $[-3S; 3S]$ ,

where  $S = 112.4$  is the corrected mean square deviation of excess. Also, 73.97% of the excess fall in the interval  $[-S;S]$ , 95.01 % are in the interval  $[-2S;2S]$ .



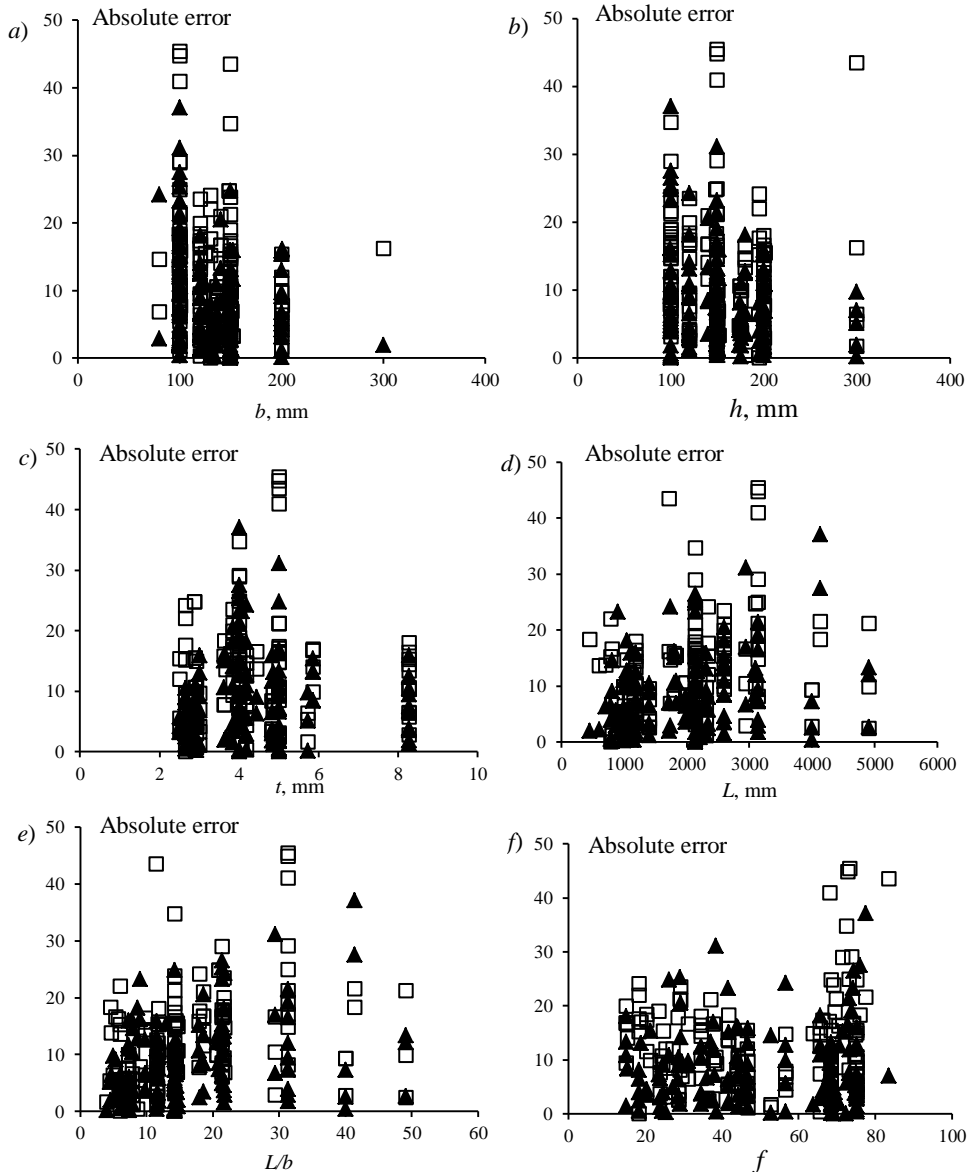
**Fig. 1.** Histogram of residuals.



**Fig. 2.** Comparison of calculation errors.

Fig. 2 shows a comparative analysis of the calculated value of the carrying capacity of rectangular columns, calculated using the presented methodology and calculation method according to Eurocode 4. In Fig. 2, triangles correspond to the error obtained by the statistical analysis methods, and squares – to those obtained according to Eurocode 4.

Figure 3 shows the dependence of the absolute error  $\Delta$  on some parameters of columns. Triangles correspond to the error obtained using the regression model, and squares – to those obtained according to Eurocode 4.



**Fig. 3.** Dependence of the regression model absolute error: a) - on  $b$ ; b) - on  $h$ ; c) on shell thickness  $t$ ; d) on the columns length  $L$ , e) on  $L/b$ , f) on concrete prism strength  $f_{c,prizm}$ .

Analysis of the factors of the regression model shows the greatest impact (79%) was exerted by the first factor  $(b - 2t)(h - 2t - e) f_{c,prizm} / E_s$ . However, to obtain a more accurate understanding of columns behaviour, other characteristics should be taken into

account (such as column length, yield strength of steel, elastic modulus of steel and concrete), as well as their combined effect. The obtained model was verified by comparing with experimental results, which were not involved in making the model [5].

Table 1 presents the comparative analysis of the obtained theoretical results ( $N_{theor}$ ) with the experimental ones ( $N_{exp}$ ), as well as with the calculations using Eurocode 4 methodologies ( $N_{Eu}$ ), Japanese ( $N_{Jpn}$ ) and Chinese ( $N_{Cn}$ ) regulatory documents [19-22].

**Table 1.** Comparative analysis.

№	$b$ , mm	$t$ , mm	$e/b$	$\sigma_y$ , MPa	$f_{c,prizm}$ , MPa	$N_{exp}$ kN	$N_{theor}$	$\Delta_{theor}$ %	$N_{Eu}$ kN	$\Delta_{Eu}$ , %	$N_{Jpn}$ kN	$\Delta_{Jpn}$ , %	$N_{Cn}$ , kN	$\Delta_{Cn}$ , %
1	160	6	0.125	352	39.4	1899	1884.45	1.39	1659	13.19	1757	8.06	1798	5.91
2						1936	1884.45	2.66	1659	14.31	1757	9.25	1798	7.13
3	160	6	0.125	361	39.4	2093	1907.35	8.87	1756	16.10	2628	25.56	1819	13.09
4						2110	1907.35	9.60	1756	16.78	2628	24.55	1819	13.79
5	160	6	0.25	360	38.1	1483	1589.83	7.20	1241	16.32	1701	14.70	1558	5.06
6						1550	1589.83	2.57	1241	19.94	1701	9.74	1558	0.52
7	160	6	0.25	351	38.1	1569	1566.86	0.14	1555	0.89	1231	21.54	1540	1.85
8						1503	1568.47	4.36	1555	3.46	1231	18.10	1540	2.46
9	160	6	0.5	360	39.5	1011	1180.89	16.80	798	21.07	887	12.27	1269	25.52
10						1080	1180.89	9.34	798	26.11	887	17.87	1269	17.50
11	160	6	0.5	355	39.5	1046	1201.24	14.84	888	15.11	1391	32.98	1261	20.55
12						1103	1201.24	8.91	888	19.49	1391	26.11	1261	14.32
13	160	6	0.75	358	35.3	727	771.34	6.10	731	0.55	905	24.48	991	36.31
14						739	771.34	4.38	731	1.08	905	22.46	991	34.10
15	160	6	0.75	352	35.3	763	792.13	3.82	825	8.13	745	2.36	983	28.83
16						793	792.13	0.11	825	4.04	745	6.05	983	23.96
17	180	8	0.125	348	55.1	3153	2799.74	10.58	2915	6.90	3799	21.34	2940	6.10
18						2976	2799.74	5.92	2915	2.05	3799	27.65	2940	1.21
19	180	8	0.25	357	50.1	2329	2465.73	5.87	1945	16.49	3071	31.86	2452	5.28
20						2489	2465.73	0.93	1945	21.86	3071	23.38	2452	1.49

### 3 Experimental tests

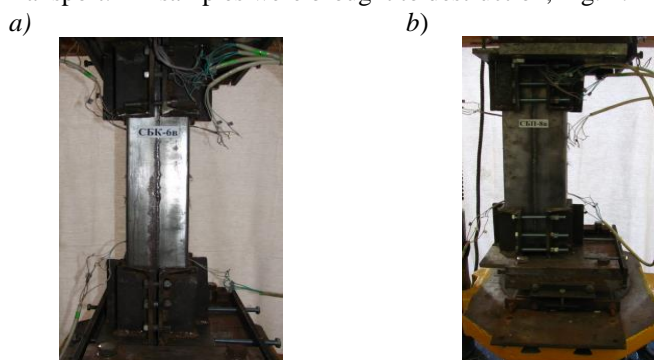
To study the nature of the destruction, experimental studies of steel concrete columns with lengths of 500, 1000, 1500 and 2000 mm were conducted. The shells of the columns had a thickness of  $t = 2$  mm, and were filled with C16/20 concrete.

The steel with a yield strength of  $\sigma_y = 240$  MPa was used to manufacture the shell. Once the metal shells were produced, they were filled with concrete mixture in an upright position with layer-by-layer compaction with a plate vibrator.

Prototypes (Fig. 4) tested for eccentric compression were labelled as follows: SBK-5v, 6v, 7v, 8v; SBK-1.0-4v, 5v, 6v; SBK-1.5-3v, 4v, 5v, 6v; SBK-2.0-3v, 4v, 5v, 6v – 100×100 mm square steel concrete columns;

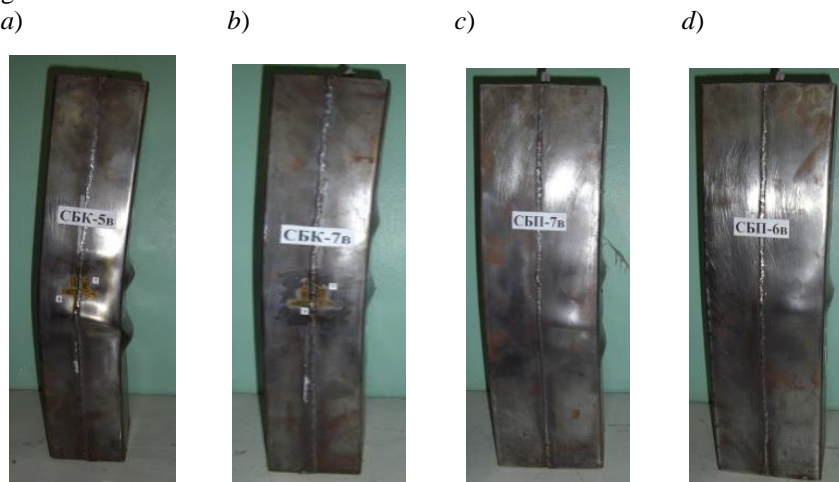
SBP-5v, 6v, 7v, 8v - 150×100 mm rectangular steel concrete columns.

Steel concrete columns were tested on IP-2000 hydraulic press in the laboratory of Structural Mechanics and Hydraulics Department of the Ukrainian State University of Railway Transport. All samples were brought to destruction, Fig. 4.



**Fig. 4.** Steel concrete columns before testing: *a)* square column, *b)* rectangular column.

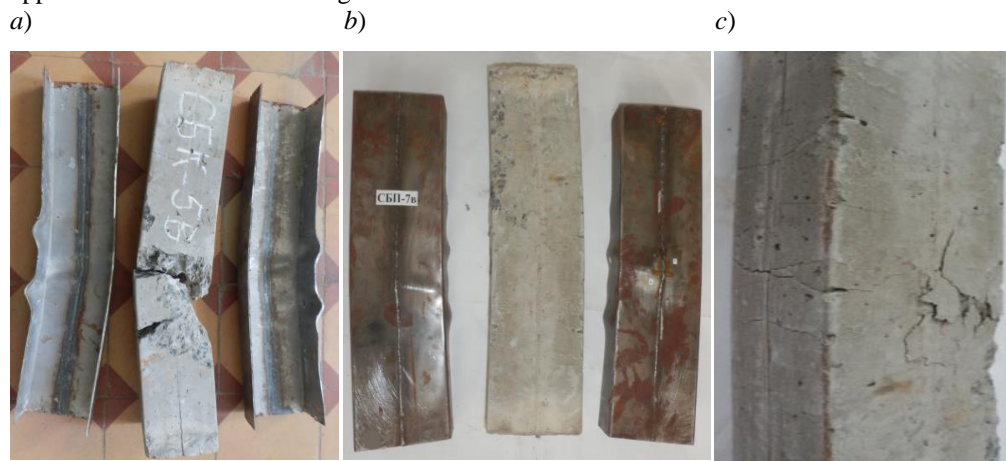
Fig. 5 shows the steel concrete columns after testing. It should be noted that the tested samples did not lose their carrying capacity instantly, as it is the case with reinforced concrete structures. After significant deformations, they still withstood a significant load for a long time.



**Fig. 5.** Steel concrete columns after testing: *a), b)* square column, *c), d)* rectangular column.

Analysis was carried out after the shell of the steel concrete columns was dismantled. The appearance of the concrete core and steel shell after dismantling is shown in Fig. 6.

Analysis of the disintegration nature of the concrete core conducted after dismantling shows that in points of local disintegration of the concrete core, concrete is detached from the shell. At this moment, a sharp redistribution of stresses occur between the components of the steel concrete element, resulting in the loss of stability of the steel shell and appearance of transverse corrugations.



**Fig. 6.** Concrete core and steel shell after dismantling the columns: a) square column, b), c) rectangular column.

The nature and type of destruction of eccentrically compressed samples were generally similar to that of centrally compressed steel concrete elements. The difference was that with eccentric application of the load, corrugations formed on the surface of the steel shell only in the maximum compression area.

At the first stage, there was elastic work of the concrete core and steel shell. The increment of the deflections and deformations of the average sectional height is practically proportional to the increase in load.

At the second (elastic plastic) stage, the shell metal in the compressed area began to yield. The dependences of deformations and deflections on the load became nonlinear. When longitudinal deformations in the steel shell reach values of about  $(1000 \div 1200) \cdot 10^{-6}$ , the elastic plastic stage of work of the samples ended.

At the third stage of (plastic) deformations of the element in the longitudinal and transverse directions increased sharply. At the plastic stage of work of steel concrete samples, most of the resistance strain gages that were in the compressed area failed due to excessive deformations of concrete and steel.

The fourth stage of work was the destruction of eccentrically compressed steel concrete elements, began with the formation of longitudinal corrugations in the maximum compression area, and ended with a sharp increase in the curvature of the element in the plane of the bending moment from the load.

As a result of the experimental tests, it was established that the longitudinal deformations of the compressed area of the shell most significantly affect the carrying capacity of the eccentrically compressed steel concrete samples. At the moment of stability loss, the shell metal reaches the yield limit only in the middle part of the compressed area.

It should be noted that in the studied steel concrete samples, the absolute magnitude of deformation of the shell of the stretched area are much lower than those of the compressed area, their values do not reach the yield limit even when samples are destroyed. So, the

compressed area is of crucial importance to characterize the stress strain state of flexible eccentrically compressed steel concrete elements.

The process of stress redistribution between the tube and concrete during testing of the steel concrete samples with a short-term load depends to a large extent on the relative length of the elements, the eccentricity of the external load, and on the magnitude of the external load.

The experimental studies carried out by the authors revealed the effect of the geometric dimensions of the steel concrete columns and physic-mechanical characteristics of the materials on the carrying capacity and deformability of columns (Table 2).

**Table 2.** Test results of steel concrete columns.

Sample No.	$L$ , mm	$e$ , mm	$N_{exp}$ , kN	Sample No.	$L$ , mm	$e$ , mm	$N_{exp}$ , kN
SBK-5v	500	25	158	SBK-1.0-6v	1000	50	79
SBK-6v	500	25	145	SBK-1.5-3v	1500	25	135
SBK-7v	500	50	82	SBK-1.5-4v	1500	25	128
SBK-8v	500	50	91	SBK-1.5-5v	1500	50	93
SBP-5v	500	50	200	SBK-1.5-6v	1500	50	89
SBP-6v	500	50	211	SBK-2.0-3v	2000	25	147
SBP-7v	500	75	155	SBK-2.0-4v	2000	25	88
SBP-8v	500	75	186	SBK-2.0-5v	2000	50	56
SBK-1.0-4v	1000	25	191	SBK-2.0-6v	2000	50	65
SBK-1.0-5v	1000	50	148				

## 4 Conclusions

1. A mathematical model has been developed to assess the effect of the geometric and physical characteristics of a rectangular column on the carrying capacity under eccentric compression.
2. High degree of model reliability is confirmed by the comparative analysis and results verification.
3. Experimental studies have been conducted to investigate the nature of the deformations development on the surface of the steel shell and inside the concrete core of rectangular columns at various stages of loading.

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