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DETERMINING ADDITIONAL POWER LOSSES IN THE ELECTRICITY SUPPLY SYSTEMS DUE TO CURRENT'S HIGHER HARMONICS

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Представлено результати досліджень впливу вищих гармонік струму навантаження на втрати потужності в електричній мережі. Актуальність цих досліджень зумовлена сталим зростанням кількості потужних імпульсних споживачів електроенергії, що призводить до збільшення вмісту вищих гармонік в системах електропостачання. В свою чергу, вищі гармоніки струму навантаження в системі електропостачання викликають погіршення не тільки якісних показників електроенергії, а й збільшення втрат потужності, що спричинено скін-ефектом в провідниках мережі та збільшенням середньоквадратичного значення струму.

Проведені дослідження показали, що існуючі аналітичні залежності активного опору мережі від частоти вищих гармонік струму під дією скін-ефекту є взаємовиключними та неточними з причини неврахування геометричних характеристик провідників мережі. На підставі рівнянь Бесселя першого роду отримано уточнені аналітичні залежності активного опору мережі від частот вищих гармонік, що враховують геометричні властивості проводів. Показано, що інтенсивність збільшення активного опору провідника відносно його опору постійного струму при заданій частоті значно залежить від діаметру провідника, що особливо важливо для ліній передачі енергії з одножильними дротами великого діаметру (наприклад, контактний дріт у залізничному електропостачанні). Показано, що для дротів малого діаметру збільшення значення активного опору під дією скін-ефекту є несуттєвим і, у такому випадку, втрати від вищих гармонік струмів зумовлені збільшенням середньоквадратичного значення струму. Представлено залежність додаткових втрат потужності в електричній мережі у функції значень коефіцієнту гармонічних спотворень струму навантаження. Приведені залежності підтверджено імітаційним моделюванням.

Отримані результати дослідження можуть бути використані при розрахунках енергетичних втрат в різноманітних електричних мережах від вищих гармонік струмів навантаження та при розрахунках економічної ефективності від впровадження фільтрокомпенсуючих пристроїв

Ключові слова: вищі гармоніки струму навантаження, втрати потужності, коефіцієнт гармонічних спотворень, скін-ефект

1. Introduction

Improving energy efficiency is a priority direction for energy generation and electricity supply systems. To achieve maximum energy effectiveness of electricity supply systems, a clear numerical understanding of the components in power losses is required, as well as methods for their elimination. In addition, one should understand the contribution of the load current higher harmonics to the loss of power, to the magnitude of total losses. This is important because the past decades have seen an increase in the number of electric energy pulse consumers, which relates both to household appliances and industrial equipment. That leads to an increase in the content of currents higher harmonics in the electricity supply systems and, consequently, to the increased power losses associated with higher harmonics, which necessitate the use of active rectifiers and active filter compensating devices [1–3]. The above predetermines the relevance of the task on defining additional power losses in the system of electricity supply due to the load currents higher harmonics.

2. Literature review and problem statement

The issue of the negative impact of higher harmonics and inactive components in load currents on the systems of electricity supply has been given considerable attention in [4–8]. However, it should be noted that the results from calculating the additional losses in the active resistance of electric networks due to the currents higher harmonics duffer significantly in a series of publications [9–13].

Additional losses of active power in a three-phase power line P_{harm}, predetermined by the passage of higher harmonics currents, are caused by the action of a skin-effect, and can be determined as the sum of losses from each harmonic:

$$P_{harm} = 3 \cdot \sum_{v=2}^{v=n} I_v^2 \cdot R_1 \cdot k_{rv}, \tag{1}$$

where v is the number of a harmonic; n is the number of considered harmonics; I_v is the rms value for the v-th harmonic; R_1 is the active resistance in a line of constant current; k_{rv} is the coefficient that takes into consideration an increase in resistance under the action of a surface effect.

However, a k_{rv} coefficient, which takes into consideration the influence of a surface effect, is determined from different expressions in publications [9–12].

Thus, paper [9] gives two different expressions to determine the dependence of coefficient k_{rv} on the order of harmonic v.

The first expression for determining the k_{rv} coefficient is as follows:

$$k_{rv} = 0{,}47 \cdot \sqrt{v}, \tag{2}$$

where $v=f/50$.

The second expression for determining the k_{rv} coefficient, in accordance with [9], depending on the frequency of the v-th harmonic is:

$$r_{0v} = r_0 \cdot (k_{pv} + k_{0v}), \tag{3}$$

where r_0 is the resistivity of an DC conductor; k_{rv} is the coefficient that takes into consideration the influence of a surface effect at the v-th harmonic; k_{ov} is the coefficient, which takes into consideration the impact of the effect of proximity for the v-th harmonic.

In accordance with [9], the k_{rv} coefficient that defines an increase in the active resistance of the conductor at the higher frequencies due to a surface effect for copper wires is:

$$k_{pv} = 0{,}021 \cdot \sqrt{f}. \tag{4}$$

For aluminum wires, k_{rv} is:

$$k_{pv} = 0{,}01635 \cdot \sqrt{f}. \tag{5}$$

The coefficient k_{ov}, which takes into consideration the effect of proximity, is determined from expression:

$$k_{0v} = \frac{1{,}18 + k_{pv}}{k_{pv} \cdot 0{,}27} \cdot \left(\frac{d}{a}\right)^2, \tag{6}$$

where d is the diameter of the conductor, mm; a is the distance between the centers of cores, mm.

In a series of studies [10–12], a k coefficient that takes into consideration an increase in active resistance under the action of a skin effect accepts a larger value and is determined from:

$$k_r = \sqrt{v}. \tag{7}$$

Article [13], based on an experimental research, gives empirical dependences of active resistance on frequency for different types of wire, as shown in Table 1.

Table 1

Approximating dependences for coefficient k_{rv2} that accounts for the impact of a skin effect for different types of cables

Conductor type	k_{rv2}
AC-400	0.3 v
A-400	0.15 v
Copper cable	0.06 v
Aluminum cable	0.06 v

Dependences of coefficients k_{rv}, k_{rv2} and k_{pv} for a copper wire on frequency are shown in Fig. 1. It should be noted that the value for coefficients k_{rv}, k_{rv2} and k_{pv} corresponds to the resistance of direct current conductors.

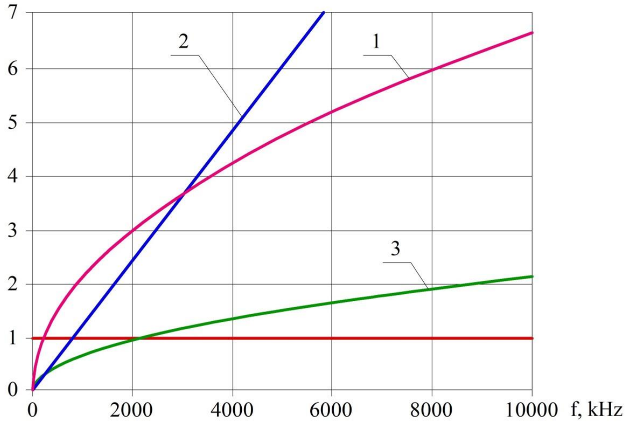

Fig. 1. Dependences of values for the coefficients that take into consideration the impact of a surface effect on an increase in active resistance, and, respectively, power losses in conductors, on the frequency of higher harmonics currents: 1 – k_{rv}; 2 – k_{rv2}; 3 – k_{pv}

Thus, our analysis leads to the conclusions on that existing dependences of active resistance in the conductors at electric networks on the frequency of higher harmonics are rather contradictory and yield different results when calculating power losses. It is worth mentioning that the specified ratios do not take into consideration the geometric properties of wires, which affect the intensity of displacement of current in a conductor during skin effect. In addition, according to expressions (2) and (4) and Table 1, coefficients k_{rv}, k_{rv2} and k_{pv}, up to a certain frequency, accept values less than unity, indicating a decrease in the network resistance relative to the resistance to direct current, that is, a certain “superconductivity” of conductors at a frequency of 50 Hz, which is false.

3. The aim and objectives of the study

The aim of this study is to derive the analytical ratios describing the energy processes related to an increase in power losses in an electric network resistance due to the current’s higher harmonics.

To accomplish the aim, the following tasks have been set:

– to derive ratios for the network active resistance due the higher harmonic frequency considering the geometrical indicators of conductors;

– to derive ratios for the additional power losses in the network resistance due to higher harmonics as a function of the current’s total harmonic distortion;

– to conduct a simulation and confirm the derived dependences of additional power losses in the network resistance due to the total harmonic distortion using a system as an example.

4. Determining the impact of a skin-effect on the network resistance based on the Bessel equations

The flow of AC current is accompanied by an electromagnetic field around the conductor, which leads to the displacement of electric charges (this same current) from the center of the conductor to its surface. This effect is called a surface effect, or the skin effect. As a result of this effect, the current density over a wire’s cross-sectional area becomes non-uniform. The volumetric density of current is maximum near the surface of the conductor. When moving away from the surface, the volumetric density decreases exponentially and, at depth Δ, it is less by e-times [14, 15]. This depth Δ is called the thickness of a skin layer and is determined from expression:

$$\Delta=\sqrt{\frac{2}{\gamma\cdot\mu\cdot 2\cdot\pi\cdot f}}, \tag{8}$$

where Δ is the depth of current penetration (m); f is the frequency of AC, Hz; γ is the specific electrical conductance, $\gamma=1/\rho$; ρ is the resistivity of the conductor ($\text{Ohm}^{-1}\cdot\text{m}^{-1}$), for copper – $1.72\cdot 10^{-8}$; for aluminum – $2.7\cdot 10^{-8}$; μ is the absolute magnetic permeability.

$$\mu=\mu_0\cdot\mu_r, \tag{9}$$

where μ_0 is the permeability of the vacuum, $\mu_0=1.25663706\times\times 10^{-6}$ N/A; μ_r is the relative permeability of a material (μ/μ_0 is the dimensionless magnitude), for copper – 0.999992; for aluminum – 1.000022.

Dependence of thickness of the skin layer of copper and aluminum wire on frequency f is shown in Fig. 2.

Thus, at a rather high frequency f=10 kHz the thickness of a skin layer becomes negligibly small (0.66 mm).

For the variable voltage a current density J from the surface to the center of the cylindrical conductor is an exponentially descending function, which is described by expression:

$$J=J_s\cdot e^{-\frac{r_k}{\Delta}}, \tag{10}$$

where J_s is the conductivity of the conductor, which corresponds to direct current; r_k is the distance from the surface of a wire to its center.

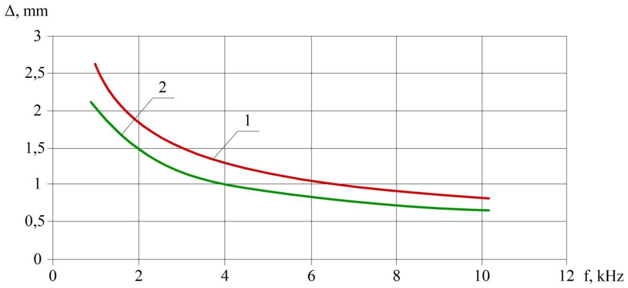

Fig. 2. Dependence of thickness of the skin layer (mm) on current frequency (kHz): 1 — for aluminum; 2 — for copper

It follows from an exponential decrease in the current density that almost all current concentrates in a layer with a thickness of several Δ. As an example, here is a distribution chart of current relative density in the conductor with a radius equal to Δ3 (Fig. 3).

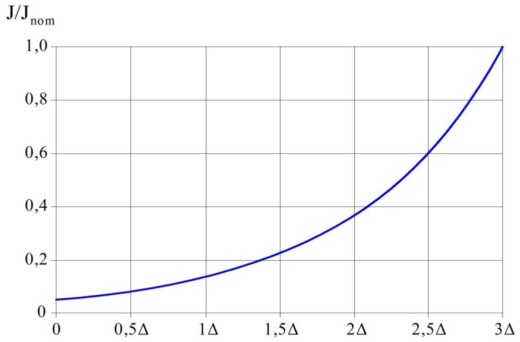

Fig. 3. Dependence of the current relative density distribution in a wire of thickness 3Δ

Thus, the current density decreases by 100 times at a depth of ≈4.6·Δ. Therefore, the impact of a skin-effect on the displacement of current is determined by the number of skin layers within the wire’s radius r_0/Δ. Such a heterogeneity in the current density leads to an increase in its specific resistance.

For the case when the radius of a wire exceeds the thickness of the skin layer, the dependence of complex resistance of wire Z on current frequency can be described by using a Bessel function [14–18], which defines the distribution of a field in the cylindrical coordinate system.

$$Z(f)=R+i\cdot X=\frac{q}{2\cdot\pi\cdot r_0\cdot\gamma}\cdot\frac{J_0\cdot(q\cdot r_0)}{J_1\cdot(q\cdot r_0)}\cdot l, \tag{11}$$

where R is the active resistance of the conductor; X is the reactive resistance of the conductor; i is the integrated component; r_0 is the radius of the wire; l is the length of the wire; $J_0(Z)$ is the Bessel function of the first kind of zero order; $J_1(Z)$ is the Bessel function of the first kind of first order; q is the wavenumber.

A wavenumber of the wire is an integrated variable and is determined based on the value for thickness of the skin layer.

$$q=k-i\cdot k;\;\; k=\frac{1}{\Delta}. \tag{12}$$

Bessel functions of the first kind are partial canonical solutions to the Bessel differential equation:

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(x^2-\alpha^2\right)=0. \qquad (13)$$

Bessel functions of the first order, denoted $J_\alpha(z)$, are the particular solutions to the Bessel differential equation for end points $x=0$ at integer and non-negative α:

$$J_\alpha(z)=\frac{z}{2}\cdot\sum_{n=0}^{n=\infty}\frac{(-1)^n}{n!(n+1+\alpha)!}\cdot\left(\frac{z}{2}\right)^{2n+\alpha}. \qquad (14)$$

The Bessel function of the first kind of zero and first order are determined as follows:

$$J_0(z)=\frac{z}{2}\cdot\sum_{n=0}^{n=\infty}\frac{(-1)^n}{n!(n+1)!}\cdot\left(\frac{z}{2}\right)^{2n}; \qquad (15)$$

$$J_1(z)=\frac{z}{2}\cdot\sum_{n=0}^{n=\infty}\frac{(-1)^n}{n!(n+2)!}\cdot\left(\frac{z}{2}\right)^{2n+1}. \qquad (16)$$

Note: solving the Bessel functions in the integrated plane is simplified by the possibility to resolve them using the software Mathcad applying the built-in functions «$J_0(z)$» and «$J_1(z)$».

Active resistance of wire R is the real part of the integrated resistance Z from expression (10). The derived dependences for the active resistance of a copper wire of length 10 km with different diameters on the frequency, calculated according to expression (11), are shown in Fig. 4.

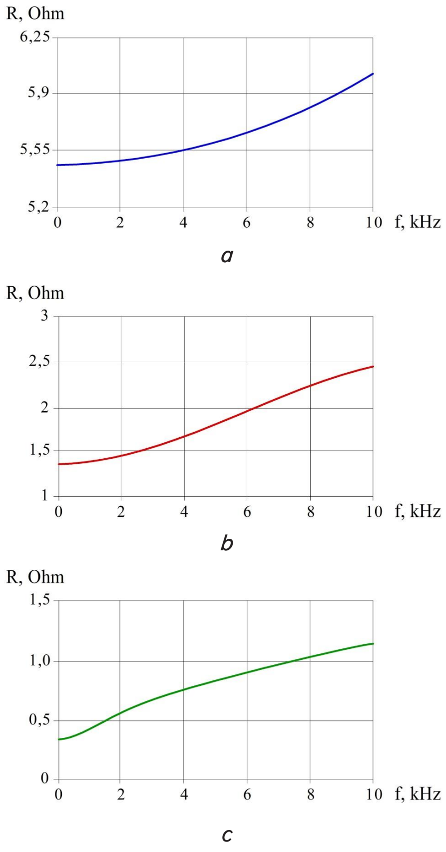

Fig. 4. Dependence of active resistances of copper wires of length 10 km with radii of 1, 2, 4 mm on frequency: *a* — 1 mm; *b* — 2 mm; *c* — 4 mm

The advantage of the proposed method for resistance calculation is the convergence between a value for the resistance at frequency close to zero and the resistance of direct current:

$$Z(f=0)=R_{DC}=\frac{L}{\gamma\cdot S}. \qquad (17)$$

The result of converting the dependences of resistances on frequency, shown in Fig. 4, into relative magnitudes, we constructed charts for the dependences of resistances in relative magnitudes (Fig. 5).

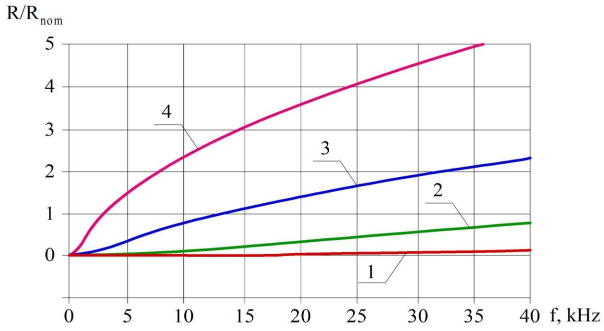

Fig. 5. Dependences of relative active resistances of copper wires with radii 1, 2, 4 mm on frequency: 1 — 0.5 mm; 2 — 1 mm; 3 — 2 mm; 4 — 4 mm

Fig. 5 shows that the diameter of the wire core largely affects the effect of displacement and, consequently, the value for active resistance at the assigned frequency.

The skin effect makes the network active resistance frequency-dependent with the network resistance growing with an increase in frequency, which leads to an increase in power losses in a power supply system. The indicated characteristics show that at the same frequency of a higher harmonic, the larger the radius of the conductor, the larger the power losses. Thus, accounting for power losses due to higher harmonics under the action of a skin-effect is particularly important for the single-core power supply systems with large radii of wires, for example, for the single-core contact wires at railroad power systems. At the same time, application of a multicore cable eliminates the negative effect of a skin-effect on losses in conductors in the electricity supply systems due to the higher harmonics of currents over a rather wide range of frequencies. However, even in the absence of the influence of a skin-effect, higher harmonics can cause an increase in additional power losses in conductors at networks due to an increase in the rms value for current, which is described in the next chapter.

5. Determining additional power losses in an electric network as a function of the total harmonic distortion in load current

We propose a method for determining additional thermal losses in electric networks of direct and alternating current due to higher harmonics, which are uniquely determined based on the resultant value for the network current total harmonic distortion. A given method can be used for the case when the effect of a skin-effect on the network resistance with a limited range of current higher harmonics is insignificant. In this case, additional losses in electric networks of alternating and direct current due to higher harmonics can be calculated based on the value for the rms value (RMS) of current, and, consequently, an increase in the losses in a quadratic dependence on the magnitude of the RMS current value.

As it is known, total harmonic distortions at direct THD_{DC} and alternating THD_{AC} currents are determined from:

$$THD_{DC} = \frac{\sqrt{\sum_{m=1}^{m=\infty} I_m^2}}{I_{DC}}; \tag{18}$$

$$THD_{AC} = \frac{\sqrt{\sum_{m=2}^{m=\infty} I_m^2}}{I_1}, \tag{19}$$

where I_m is the RMS value for the m-th harmonic; I_{DC} is the value for a direct component.

For the subsequent formulae, a value for THD is given in relative magnitudes, that is, from 0 to 1.

As is known, the acting value (it is the same as RMS value) for alternating (or direct pulsed) current is equal to the magnitude of such a direct current, which, over the time equal to a single period of AC, would produce the same work (heat or electrodynamic effect) that would be produced by the examined alternating current.

$$I_{RMS} = \sqrt{\frac{1}{T} \cdot \int_0^T i^2(t) \cdot \mathrm{d}t}. \tag{20}$$

AC rms value can also be expressed through a range of higher harmonics:

$$I_{RMS\_AC} = \sqrt{I_1^2 + \sum_{m=2}^{m=\infty} I_m^2}. \tag{21}$$

Using expression (17), one can express the sum of squares of higher harmonics:

$$\sum_{m=2}^{m=\infty} I_m^2 = (THD_I \cdot I_I)^2. \tag{22}$$

Then the rms value for direct and alternating currents can be represented in the form:

$$I_{RMS\_AC} = \sqrt{I_1^2 + (THD_I \cdot I_1)^2} = \sqrt{I_1^2 \cdot (1 + THD_I^2)}; \tag{23}$$

$$I_{RMS\_DC} = I_{DC} \cdot \sqrt{(1 + THD_{DC}^2)}. \tag{24}$$

The dependence of rms current value on THD value is shown in Fig. 6, where 100 % is accepted to be a rms value for the current first harmonic.

Thus, there is a clear relationship between the total harmonic distortion of the current used and the percentage of additional power losses.

The dependence of relative value for additional losses on value for the total harmonic distortion is shown in Fig. 7, where 100 % is accepted to be the losses due to the main harmonic, or for DC network of direct current component.

The ratios that are shown in Fig. 7 make it possible to determine additional losses in system of electric supply on the value for total harmonic distortion (THD) of load current. It follows from Fig. 7 that the distortion of network current at the total harmonic distortion of 50 % causes an increase in power losses in an electric network by about 25 %.

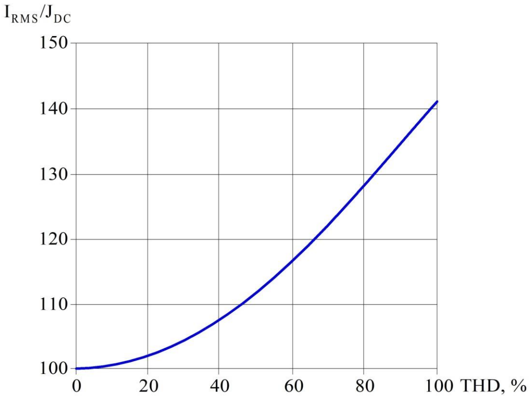

Fig. 6. Dependence of the current rms relative value on the total harmonic distortion

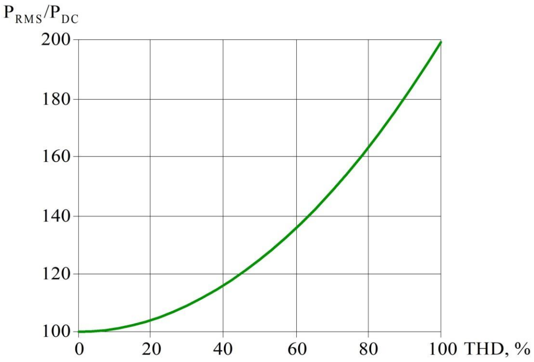

Fig. 7. Dependence of the percentage value for relative additional losses on the total harmonic distortion

6. Simulation analysis of additional losses in active resistance of electric network due to the current higher harmonics

To confirm the dependences represented in the preceding chapter, we performed a simulation in the programming environment to investigate power losses in the electric network resistance due to the higher harmonics in a three-phase diode rectifier and a decrease in losses when enabling a power active filter (PAF). A simulation model is shown in Fig. 8.

Parameters for the simulation model: network power voltage is 3x380 V; active network resistance is 0.2 Ohms, network inductance is 50 μGn, inductance of PAF throttles is 5 mGn; inductance of the input choke of the diode rectifier is 0.1 mGn; capacitance of the rectifier filter is 12 mF; resistance in the rectifier load is – 3 Ohms. The maximum relative error of calculations in the software Matlab is 0.1 %.

Control system of the power active filter is based on the principle of hysteresis modulation and the power pqr-theory. PAF operates under a mode of counter-phase generation of the load current higher harmonics. That ensures mutual compensation of the load current higher harmonics by the current of PAF, which makes it possible to provide for the network current THD below 3 %. The principles of the power pqr-theory and description of the control system, as well as electrical processes at a power active filter, are given in more detail in [18–22].

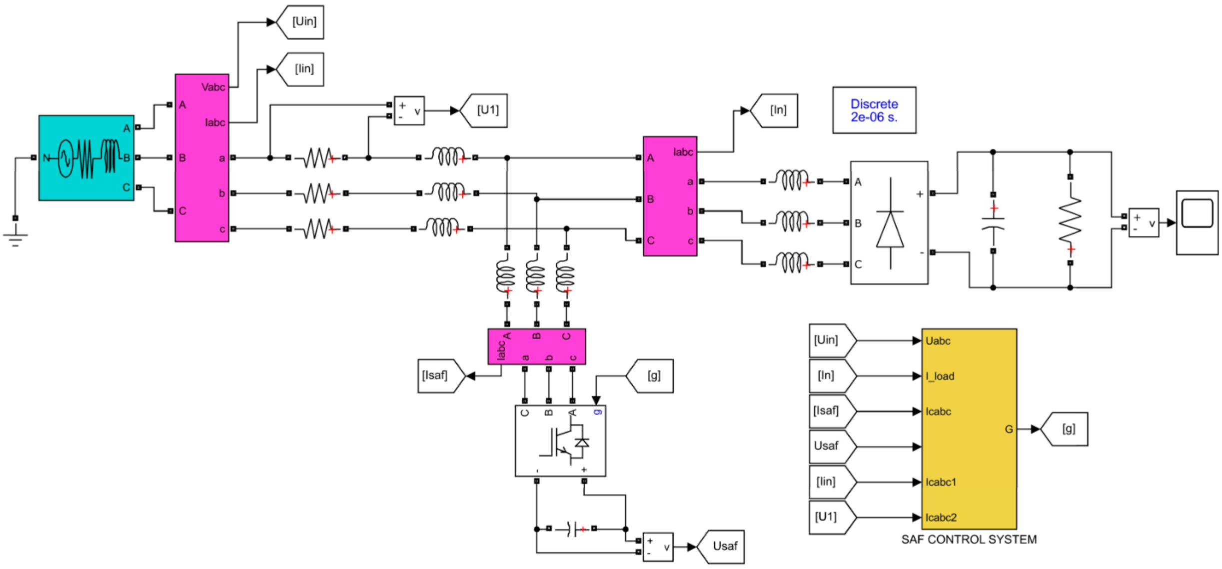

Fig. 8. Simulation model of electric network with a three-phase diode rectifier and a power active filter

The results of simulation, namely the input current in a diode rectifier, the current generated by PAF, and the resulting current in the network, are shown in Fig. 9.

Results of the Fourier analysis into a network current with and without PAF, performed in the software Matlab, are shown in Fig. 10.

Numerical results from simulation are given in Table 2.

Results of simulation demonstrate that the use of a power active filter makes it possible to significantly reduce the content of higher harmonics in the network, reduce the rms value for network current, and, consequently, losses in the active resistance of electric network. The simulation showed that the use of a power active filter makes it possible to reduce losses in the network active resistance for a particular consumer from 3.56 kW to 2.83 kW, which coincides with the analytical ratios shown in Fig. 7.

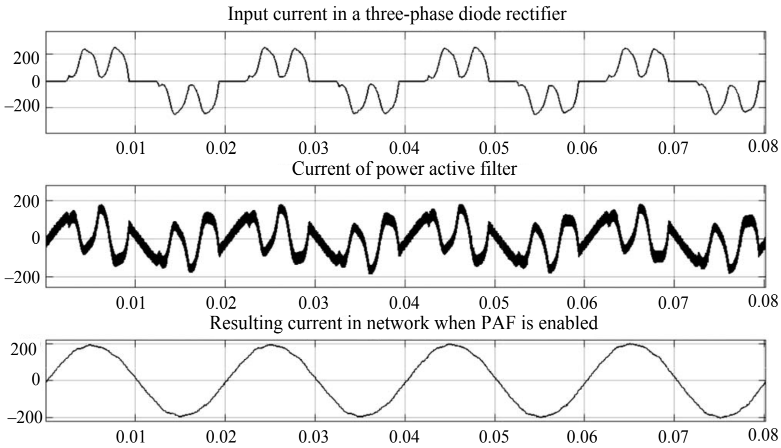

Fig. 9. Oscillograms acquired from simulation modeling: input current in a three-phase diode rectifier (without PAF), the current generated by PAF, the resulting network current when using PAF

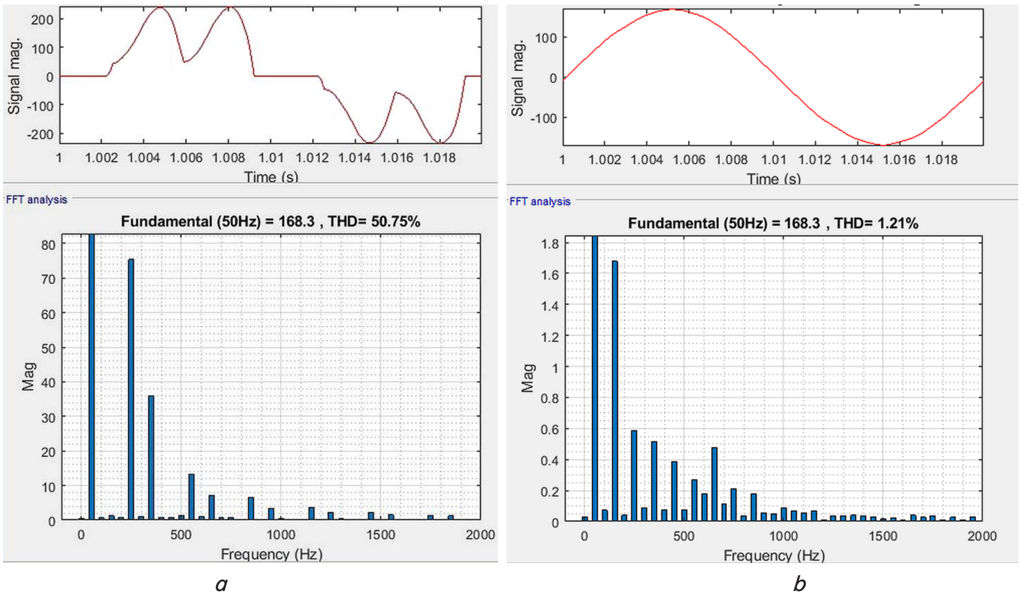

Fig. 10. Results of the Fourier analysis: *a* — input current in a diode rectifier; *b* — network current when using PAF

Table 2

Results of simulation

Parameter	Value prior to filtration	Value upon filtration
First harmonic of network current, A	119.006	119.006
Network current total harmonic distortion THD, %	50.75	1.21
Rms value for network current, A	133.45	119.014
Power of losses in network active resistance, caused by first current harmonica, kW	2.8324	2.8324
Total power of losses in network active resistance, kW	3.5617	2.8332

7. Discussion of results of examining the analytical ratios

An increase in the power losses in the network active resistance due to the load current higher harmonics is determined as the sum of power losses due to each harmonic. Under the action of a skin-effect, an increase in the frequency of higher harmonics leads to an increase in the network active resistance. The existing ratios that determine the value for active resistance due to the frequency of higher harmonics are mutually exclusive. In addition, for the range of low frequencies these expressions are false, because they determine the value of network active resistance for a current with a frequency of 50 Hz and for a harmonic of 150 Hz, which is much less than for DC.

In the present work, we derived analytical dependences for active resistance of wires in a power network on the frequency of harmonic spectrum of load current. A special feature of this work is the application in the course of calculating the network electric resistance of Bessel equations. The advantage of the proposed equations, in comparison with known equations, is the consideration of geometrical parameters of electrical conductors (length and radius), which greatly affect the intensity of the skin effect and a change in active resistance. It is shown that the influence of the skin-effect on power losses is of great importance when using single-core wires of a large diameter, as, for example, for the contact network during railroad transportation. For the case of using multi-core wires of a small radius at a range of higher harmonics limited to the dozen kHz, the impact of the skin effect significantly decreases, but in this case one should additionally take into consideration the proximity effect, determining the influence of which is important for the further studies. When advancing the present research, one needs to conduct physical experiments that would confirm the represented expressions for the influence of the skin-effect on active resistance. The process of verification is complicated because of the fact that when conducting them such simulation tools as Matlab, Multisim, MicroCAP, P-Spice and others do not take into consideration a change in the electrical network resistance due to the frequencies of current higher harmonics under the action of the skin effect.

For the case when the range of higher harmonics is limited and an increase in the network active resistance at this frequency range is insignificant, the influence of the skin effect can be neglected. In this case, the impact of load current higher harmonics on power losses in the electrical network can be determined based on the rms value for a load current. This work provides analytical dependences for a relative increase in the losses of power due to the network current total harmonic distortion. The dependences given have been confirmed in the course of simulation in the Matlab environment using an example of operation of the system of three-phase power supply with a diode rectifier and a power active filter.

Our research results could be practically applied:

– when calculating power losses in electricity supply systems, AC and DC;

– when estimating power losses in an electric network from a particular consumer, as a factor in feasibility study into the application of power active filters and other filter compensating devices that are designed to reduce the content of higher harmonics.

8. Conclusions

1. We have determined the ratio for dependence of the network active resistance as a function of the higher harmonics' frequency and the geometrical characteristics of conductors (length and diameter). A series of dependences were established for the active resistance of wires of different radii on the frequency of higher harmonics (Fig. 5). For example, it is shown that at a wire's radius of 4 mm, for the harmonic of current with a frequency of 8 kHz, an increase in the network active relative to the resistance of direct current increases by almost 100 %, which, consequently, leads to the proportional increase in power losses due to this harmonic.

2. We have established a dependence of additional power losses in the network active resistance on higher harmonics as a function of the load current total harmonic distortion. It is shown that in the range of the input current THD values from 30 % to 80 % additional losses in an electric network would grow, accordingly, from 10 % to 48 % relative to the electrical resistance of the direct current conductor.

3. The simulation conducted has confirmed a decrease in losses in the network active resistance at a decrease in the composition of higher harmonics using an example of a power network with a three-phase diode rectifier and a power active filter. Simulation has confirmed that a 50.75 % THD of input current in a diode rectifier predetermined 25.78 % of additional losses in the active resistance of a power network.

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