Calculation of Critical Loading of Beam Columns

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Abstract: This article shows the dependences connecting the deflection, a critical force and the location of the dangerous section. The following design schemes are being considered: a column, loaded at the ends of the longitudinal force equal to one side and pointing eccentricities; a colon which is rigidly clamped bottom and hinged top, eccentrically loaded longitudinal force applied on the upper end; columns, rigid clamping from below, at the free end loaded eccentrically applied force; a column, hinged, loaded on the upper end of the eccentrically attached to the longitudinal force. As in some of the equations the function is not defined, then we will not be able to determine the value of critical force analytically, but knowing the geometric characteristics of the column and the allowable deflection, we can calculate the value of the critical force.
Keywords: column; eccentrically compression; critical load; dangerous section; deflection.

1. Introduction

Vatulya G. [5], Glazunov Y, Zhakin I., Opanasenko E., Storozhenko L., Chikhladze E. [3, 4, 5] and other scholars dedicated their works to the experimental and theoretical investigations of stress-strain state of the central and eccentrically compressed columns and other supporting structures. The marked works have contributed to the solution of the difficult problem of assessing the strength and stiffness of the columns. However, not all issues have been resolved. In particular: the work of statically determinate and statically indeterminate columns compressed eccentrically at one end and such that the axial compressions test - on the other has not been studied thoroughly. In this regard, this article provides solutions that allow you to determine the critical load of the following design schemes (Fig. 1): columns, loaded at the ends of the longitudinal force equal to one side and pointing eccentricities; a colon rigidly clamped bottom and hinged top, eccentrically loaded longitudinal force applied on the upper end; columns, rigid clamping from below, at the free end loaded eccentrically applied force; columns, hinged, loaded on the upper end of the eccentrically attached to the longitudinal force.

2. The formulation of the problem

To determine the critical load eccentrically compressed columns at first it is necessary to get the relation between load and deflection which occur in columns under various loading schemes.

2.1 The column is loaded with two compressive forces applied eccentrically

Consider a column loaded with two compressive forces applied to the eccentricity e (Fig. 1a). The differential equation of the deflected axis will have the following form:

$$y'' + k^2 \cdot y = -k^2 \cdot e \tag{1}$$

where:

$$k^2 = \frac{F}{EI} \tag{2}$$

Solution of equation (1):

$$y = e\left(\cos kx + \frac{1-\cos kl}{\sin kl}\sin kx - 1\right) \tag{3}$$

The cross-section with a maximum deflection is at a distance $x = 0.5l$ from the origin.

The maximum deflection is defined by the formula:

$$y = e\left(cos0.5kl + \frac{1-coskl}{sinkl}sin0.5kl - 1\right) \tag{4}$$

Substitute equation (2) into the equation (4) and we get the following

$$y = e\left(cos\left(0.5l\sqrt{\frac{F}{EI}}\right) + \frac{1-cos\left(l\sqrt{\frac{F}{EI}}\right)}{sin\left(l\sqrt{\frac{F}{EI}}\right)}sin\left(0.5l\sqrt{\frac{F}{EI}}\right) - 1\right) \tag{5}$$

Use a computer program to express the value of the critical force from the equation (5)

$$F = \frac{4\,arctg\left(\frac{e\cdot\sqrt{2ey+y^2}}{y+e}\right)^2 EI}{l^2} \tag{6}$$

Substitute the value of the allowable deflection of the column, according to [7], we find the critical force.

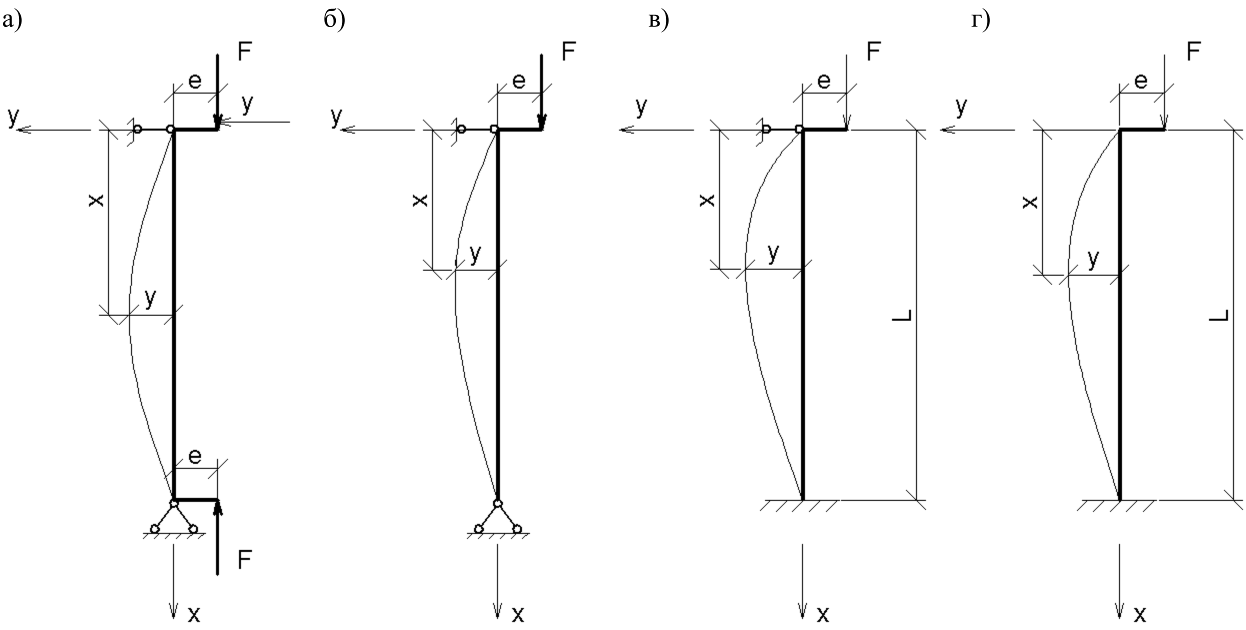

Fig. 1 Schemes of loading columns

2.2 A hinged column, loaded with force F, which is attached eccentrically on the top

Consider a hinged column loaded by force F, which is attached with eccentricity at the upper end (Fig. 1b). The differential equation of the curved axis will have the following form:

$$y'' + k^2 \cdot y = \frac{k^2 \cdot e \cdot x}{l} - k^2 \cdot e \tag{7}$$

solution of equation (7):

$$y = e \cdot \left[\cos kx - \mathrm{ctg} kl \cdot \sin kx - 1 + \frac{x}{l}\right] \tag{8}$$

That section will be dangerous which will have the maximum deflection. To determine the dangerous section it is necessary to find the point of extreme (8). Use the computer program where you will find a cross section of maximum deflection

$$x = \frac{2\mathrm{arctg}\left(\dfrac{kl \cdot \mathrm{tg} kl - \sqrt{\dfrac{k^2 l^2 - 1}{\cos(kl)^2} + 1}}{kl + \mathrm{tg} kl}\right)}{k} \tag{9}$$

Substitute equation (2) and the equation (9) into equation (8), and we get

$$\begin{aligned} y = {} & e \cdot \cos\left(2\,\mathrm{arctg}\left(\frac{\sqrt{\frac{F}{EI}}\,l \cdot \mathrm{tg}\sqrt{\frac{F}{EI}}\,l - \sqrt{\dfrac{\frac{F}{EI}l^2 - 1}{\cos\left(\sqrt{\frac{F}{EI}}\,l\right)^2} + 1}}{\sqrt{\frac{F}{EI}}\,l + \mathrm{tg}\sqrt{\frac{F}{EI}}\,l}\right)\right) - \\ & -e \cdot \mathrm{ctg}\sqrt{\frac{F}{EI}}\,l \cdot \sin\left(2\,\mathrm{arctg}\left(\frac{\sqrt{\frac{F}{EI}}\,l \cdot \mathrm{tg}\sqrt{\frac{F}{EI}}\,l - \sqrt{\dfrac{\frac{F}{EI}l^2 - 1}{\cos\left(\sqrt{\frac{F}{EI}}\,l\right)^2} + 1}}{\sqrt{\frac{F}{EI}}\,l + \mathrm{tg}\sqrt{\frac{F}{EI}}\,l}\right)\right) - \\ & -e + e\,\frac{2\mathrm{arctg}\left(\dfrac{\sqrt{\frac{F}{EI}}\,l \cdot \mathrm{tg}\sqrt{\frac{F}{EI}}\,l - \sqrt{\dfrac{\frac{F}{EI}l^2 - 1}{\cos\left(\sqrt{\frac{F}{EI}}\,l\right)^2} + 1}}{\sqrt{\frac{F}{EI}}\,l + \mathrm{tg}\sqrt{\frac{F}{EI}}\,l}\right)}{\sqrt{\frac{F}{EI}}\,l} \end{aligned} \tag{10}$$

As in the equation (10) the function is not defined then, we will not be able to define the value of the critical force analytically but knowing the geometric characteristics of the column and the allowable deflection, we can get the value of the critical force.

2.3 A column rigidly clamped at the bottom, hinged top, eccentrically loaded longitudinal force applied on the top

Consider a column of rigid bottom hinged top loaded eccentrically by a longitudinal force applied at the upper end (Fig. 1c). The differential equation of the deflected axis will have the following form:

$$y'' + k^2 \cdot y = \frac{k^2 \cdot e \cdot x}{2 \cdot l} - k^2 \tag{11}$$

solution of equation (11):

$$y = e\left(\cos kx - \left(\frac{\cos kl + 0.5}{\sin kl}\right) \cdot \sin kx - 1 + \frac{x}{2 \cdot l}\right) \tag{12}$$

That section will be dangerous which will have the maximum deflection. To determine the dangerous section it is necessary to find the point of extreme (12). Use the computer program where you will find a cross section of maximum deflection

$$x = \frac{1}{k} \cdot 2\mathrm{arctg}\Bigg(\frac{-4kl\,\mathrm{tg}(0.5kl)}{kl\,\mathrm{tg}(0.5kl)^2 - 3kl - 6\mathrm{tg}(0.5kl)} + \\ + \frac{\sqrt{k^2l^2\mathrm{tg}(0.5kl)^4 + 10k^2l^2\mathrm{tg}(0.5kl)^2 + 9k^2l^2 - 36\mathrm{tg}(0.5kl)^2}}{kl\,\mathrm{tg}(0.5kl)^2 - 3kl - 6\mathrm{tg}(0.5kl)}\Bigg) \tag{13}$$

Substitute the equation (2) and the equation (13) into the equation (12) and we get the equation (14), in which the function is not defined and we will not be able to define the value of the critical force analytically but knowing the geometric characteristics of the column and the allowable deflection, we can get the value of the critical force.

$$y = e \cdot \cos\left(2\mathrm{arctg}\left(\frac{l\sqrt{\frac{F}{EI}}\,\mathrm{tg}\left(l\sqrt{\frac{F}{EI}}\right) - \sqrt{\frac{Fl^2}{EI\cos\left(l\sqrt{\frac{F}{EI}}\right)^2} + 1 - \frac{1}{\cos\left(l\sqrt{\frac{F}{EI}}\right)^2}}}{l\sqrt{\frac{F}{EI}} + \mathrm{tg}\left(l\sqrt{\frac{F}{EI}}\right)}\right)\right) - \\ - e\,\mathrm{ctg}\left(l\sqrt{\frac{F}{EI}}\right)\sin\left(2\mathrm{arctg}\left(\frac{l\sqrt{\frac{F}{EI}}\,\mathrm{tg}\left(l\sqrt{\frac{F}{EI}}\right) - \sqrt{\frac{Fl^2}{EI\cos\left(l\sqrt{\frac{F}{EI}}\right)^2} + 1 - \frac{1}{\cos\left(l\sqrt{\frac{F}{EI}}\right)^2}}}{l\sqrt{\frac{F}{EI}} + \mathrm{tg}\left(l\sqrt{\frac{F}{EI}}\right)}\right)\right) - \\ - e + e\,\frac{2\mathrm{arctg}\left(\frac{l\sqrt{\frac{F}{EI}}\,\mathrm{tg}\left(l\sqrt{\frac{F}{EI}}\right) - \sqrt{\frac{Fl^2}{EI\cos\left(l\sqrt{\frac{F}{EI}}\right)^2} + 1 - \frac{1}{\cos\left(l\sqrt{\frac{F}{EI}}\right)^2}}}{l\sqrt{\frac{F}{EI}} + \mathrm{tg}\left(l\sqrt{\frac{F}{EI}}\right)}\right)}{l\sqrt{\frac{F}{EI}}} \tag{14}$$

2.4 Column rigidly clamped below, is loaded at the free end of the longitudinal force applied eccentric

Column rigidly clamped from below, at the free end is loaded eccentrically applied force (Fig. 1d). The differential equation of the deflected axis will have the following form:

$$y'' + k^2 y = -k^2 e \tag{15}$$

solution of equation (15):

$$y = e\left(\cos kx + \frac{1 - \cos kl}{\sin kl}\sin kx - 1\right) \tag{16}$$

That section will be dangerous which will have the maximum deflection. To determine the dangerous section it is necessary to find the point of extreme (16). Use the computer program where you will find a cross section of maximum deflection

$$x = -\frac{\operatorname{arctg}\left(\frac{\cos(0.5kl) - 1}{\sin(0.5kl)}\right)}{k} \tag{17}$$

Substitute the equation (2) and the equation (17) into the equation (16) we get

$$\begin{gathered} y = e \cdot \cos\left(-\operatorname{arctg}\left(\frac{\cos\left(0.5l\sqrt{\frac{F}{EI}}\right) - 1}{\sin\left(0.5l\sqrt{\frac{F}{EI}}\right)}\right)\right) + \\ + e \cdot \left(\frac{1 - \cos l\sqrt{\frac{F}{EI}}}{\sin l\sqrt{\frac{F}{EI}}}\right)\sin\left(-\operatorname{arctg}\left(\frac{\cos\left(0.5l\sqrt{\frac{F}{EI}}\right) - 1}{\sin\left(0.5l\sqrt{\frac{F}{EI}}\right)}\right)\right) - e \end{gathered} \tag{18}$$

Use a computer program to express the value of the critical force from equation (18).

$$F = \frac{4\operatorname{arctg}\left(-\frac{2e \cdot \sqrt{2ey + y^2}}{(y + e)^2}\right)^2 EI}{l^2} \tag{19}$$

Substitute the value of the allowable deflection of the column, according to [7], we find the critical force.

3. Conclusions

The obtained solutions allow us to determine the critical load of the following calculation schemes: columns, loaded at the ends of the longitudinal force equal to one side and pointing eccentricities; Colon rigidly clamped bottom, hinged top, eccentrically loaded longitudinal force applied on the upper end; columns, rigid clamping from below, at the free end loaded eccentrically applied force; columns, hinged, loaded on the upper end of the eccentrically attached to the longitudinal force.

4. References

[1] Trulli, V.A. To the question of calculation of stability is compressed-bent rods with an approximate expression for the curvature. [Text]: Vol. Research on the building design and testing facilities. / V.A. Trull RS Sanzharovsky, RS: L., 1968. - S.23-25. (In Russian).

[2] Hoff, N.D. Buckling and stability [Text]: Vol. / N.D.Hoff M.: Publishing House of Foreign. Lighted, 1955. - 155 p. (In Russian).

[3] Chikhladze E.D .Calculation of steel-concrete elements of rectangular cross section for durability under eccentric compression and bending // Tr. in. - And HIIT. 1993. Vyp.21. - S.23-25. (In Russian).

[4] Chikhladze E.D., Arslankhanov AND Calculation of steel-concrete elements of rectangular cross section for durability under eccentric compression and bending // Proceedings of the universities. Building. 1992. №1. - S.6-10. (In Russian).

[5] Basis of calculation and design of composite steel-concrete structures and / [Chikhladze ED, Vatulya G.L., whales, JP et al.]; ed E.D. Chikhladze - Kiev: Transport of Ukraine, 2006. - 136 p. (In Russian).

[6] Galagurya, E.I. Test of steel and steel-concrete box-section columns on eccentric and axial compression [Text]: ST. Sciences. Works / Interagency nauk. Techno. ST. Sciences. Works. State Scientific Research Institute of building structures of the Ministry of Regional Development and Construction of Ukraine. - Kiev: Publishing NDIBK, 2007. - Vip. 67 - S. 112 - 118. (In Russian).

[7] DSTU B. V.1.2-3: 2006. Deflections and moving. Requirements engineering [Text]: To replace Snip 2.01.07-85; valid from 2007 01 01. ¬ K.: State Standard of Ukraine, 2007.¬ 14c. (In Ukrainian).

[8] DSTU B. V.2.6-75: 2008. Construction of houses and buildings. Construction metal building. General specifications [Text]: GOST 23118-78 replaced; in force since 2010 01 01. ¬ K.: State Standard of Ukraine, 2008.¬ 13c. (In Ukrainian).

[9] Steel construction. The rules of designing, manufacturing and installation [Text]: DBN The 2.6-163: 2010: SHUTTER. Minregionbud Ukraine 30.12.2010. - K.: Minregionbud Ukraine 2010.-201c (State building codes Ukraine). (In Ukrainian).